Guidelines for the Final Project

Your final project can consist of a philosophical discussion of some issue connected to the material we have discussed in the course or, if you prefer, can present a formal result of interest to you. If you choose a philosophical project, your discussion should approach 10 to 15 pages, whereas a technical project should not involve more than 5 to 8 pages.

The final project will be due at midnight on Wednesday, 12/10. To make sure you are on track to complete the project in time, you should plan to have a bibliography and extended abstract by Monday, November 17.

What follows is a list of sample topics that would be appropriate for a final project for the course. The list is by no means exhaustive. Please feel free to find an alternative project in the area, and I would be happy to discuss it with you. In the meantime, I hope to add more suggestions as we cover more ground.

Philosophical Projects

On Metaphysical Necessity

1. What is the modal logic of metaphysical necessity? Both S4 and S5 have been proposed as candidates, but they remain controversial.
   - One influential argument against the validity of axiom 4 when $\Box$ is interpreted in terms of metaphysical modality is discussed by Nathan Salmon in "The Logic of What Might Have Been," *Philosophical Review,* 1989, 98, 3-34. In that paper, he argues that axiom B may be true, but neither analytic nor a truth of logic.
   - One recent response to this argument is discussed by Sarah-Jane Leslie in "Essence, Plenitude, and Paradox," *Philosophical Perspectives* 2011, 25, 277-295.
2. A concise formulation of the *modal ontological argument* is as follows:
   (a) Necessarily, if God exists, then God necessarily exists.
   (b) God possibly exists.
   (c) Therefore, God exists.

   When formalized in the language of modal propositional logic, the argument becomes valid only against the background of certain powerful normal modal systems such as $S5$.

   On the other hand, there appears to be a parallel argument available for the non-existence of God, which may be paraphrased as follows:
   (a) Necessarily, if God exists, then God necessarily exists.
   (b) God possibly does not exist.

(c) Therefore, God does not exist.

The argument has recently been called the *reverse modal ontological argument* for atheism. The symmetry problem is the observation that the standard reasons offered in support of the second premise of the modal ontological argument appear to support the second premise of the reverse ontological argument. Here are some important questions to consider:

- Which modal logics validate the modal ontological argument?
- Which modal logics validate the reverse modal ontological argument?
- Is there a principled reason to prefer one argument over the other?

Both arguments have recently been discussed in Fritz, Lo, Schmidt, "Symmetry Lost: A Modal Ontological Argument for Atheism," forthcoming in *Nous*.

On Tense and Choice

3. Fatalism is the thesis that there is nothing we can do to influence the future. One way to motivate the view is through the argument below:

   (a) Either I will φ tomorrow or I will not.

   (b) If I will φ tomorrow, then it has always been the case that I will φ tomorrow.

   (c) If it has always been the case that I will φ tomorrow, then I will necessarily φ.

   (d) If I will not φ tomorrow, then it has always been the case that I will not φ tomorrow.

   (e) If it has always been the case that I will not φ, then necessarily, I will not φ tomorrow.

   (f) I will necessarily φ tomorrow or I will necessarily not φ tomorrow.

   Explain how to expand the framework of tense logic to present and respond to the argument.

On Epistemic Modalities

4. Robert Stalnaker has derived characterization of belief in terms of ignorance in "On Logics of Knowledge and Belief," *Philosophical Studies*, 2006: 169-199. His framework codifies a battery of principles on the interaction between knowledge and belief. For Stalnaker, to believe a proposition is to regard it as open that one knows that proposition. Otherwise put, to believe is to not be able to rule out one knows.

   - How plausible is Stalnaker's characterization of belief in terms of knowledge?

- How sensitive is the characterization to specific choices for the logic of knowledge and belief?

On Counterfactuals

5. Kit Fine has raised a challenge for possible worlds treatments of counterfactuals in the style of Lewis and Stalnaker in "Counterfactuals without Possible Worlds," *Journal of Philosophy* 109 (3): 221-246.
   - Explain the nature of the difficulty Fine has isolated.
   - What is the best response on behalf of possible worlds approaches to counterfactuals?
6. Conditional Excluded Middle (CEM) is the principle:

$$(p > q) \vee (p > \neg q) \tag{CEM}$$

   The validity of CEM in Stalnaker's logic is part of the reason Lewis to part ways with Stalnaker. Explain what is at stake in the debate over CEM. There are several aspects to this question:
   - CEM ruins a suggestive analysis of *might* in terms of *would*:

$$p \mathrel{\Diamond\!\!\rightarrow} q := \neg(p \mathrel{\Box\!\!\rightarrow} \neg q)$$

   - CEM looks quite implausible for counterfactuals. The following counterfactuals seem false for a coin that is not tossed:
     - If the coin had been tossed, it would have landed heads.
     - If the coin had been tossed, it would have not landed heads – and thus it would have landed tails.

     What appears to be true instead is the following:
     - If the coin had been tossed, it might have landed heads.
     - If the coin had been tossed, it might have landed tails.

     How should a proponent of CEM explain these judgments?

On Quantification and Modality

7. The simplest quantified modal logic combines the axioms of pure quantificational logic with identity with the axioms of S5. Let us interpret the language of quantified modal logic with the help of a constant domain possible worlds model theory.
   - Outline a derivation of the necessary necessity of existence (NNE) as a theorem: $\Box\forall x\Box\exists y\ x = y$.

- Outline a derivation of the Converse Barcan Formula (CBF) as a theorem of the system: $\Box\forall x\varphi \rightarrow \forall x\Box\varphi$.
- Discuss the status of the Barcan Formula (BF) in the framework: $\forall x\Box\varphi \rightarrow \Box\forall x\varphi$.
- What is the ontological significance of these results?

8. In "Semantical Considerations on Modal Logic," Saul Kripke offers an axiomatization of quantified modal logic, which combines the axioms of $S5$ with a free axiomatization of pure quantificational logic with identity. He interprets the language with the help of a variable domain possible worlds model theory.
   - Explain how Kripke proposes to block the proof of the Converse Barcan Formula (CBF) in his system: $\Box\forall x\varphi \rightarrow \forall x\Box\varphi$.
   - Discuss the status of the necessary necessity of existence (NNE) in Kripke's system: $\Box\forall x\Box\exists y\ x = y$.
   - What exactly does it take to verify all instances of each the Converse Barcan Formula (CBF) and the Barcan Formula (BF) in a model of a variable domain possible worlds model theory?
   - Explain how to articulate the contingency of existence in Kripke's framework.

Technical Projects

1. Find a modal logic that characterizes the class of finite transitive trees. Please note that the modal logic in question may not be canonical in which case the justification of completeness may require a different method. Hints available upon request.
2. Consider the tense logic that results when we supplement K_t with all substitution instances of the axioms:

   $$\mathsf{F}q \wedge \mathsf{F}q \rightarrow (\mathsf{F}(p \wedge \mathsf{F}q) \vee \mathsf{F}(p \wedge q) \vee \mathsf{F}(\mathsf{F}p \wedge q)$$
   $$\mathsf{G}p \rightarrow \mathsf{F}p$$
   $$\mathsf{H}(\mathsf{H}p \rightarrow p) \rightarrow \mathsf{H}p$$
   $$\mathsf{GF}p \rightarrow \mathsf{FG}p$$

   Justify the following two claims:
   - The tense logic above defines the empty class of frames.
   - The tense logic above is consistent and therefore incomplete with respect to the empty class of frames.
3. Robert Stalnaker considers a definition of belief as a species of ignorance in "On Logics of Knowledge and Belief," *Philosophical Studies*, 2006: 169-199.

- Explain how to recover the axioms of the logic of belief KD45 from Stalnaker's definition of belief in terms of knowledge against the background of the logic of knowledge S4.2. Stalnaker's definition of belief is:

$$\mathsf{B}_i\varphi := \langle \mathsf{K}_i \rangle \mathsf{K}_i \varphi.$$

The axioms of the logic of knowledge S4.2 Include:

$\mathsf{K}_i(\varphi \to \psi) \to (\mathsf{K}_i\varphi \to \mathsf{K}_i\psi)$	K
$\mathsf{K}_i\varphi \to \varphi$	T
$\mathsf{K}_i\varphi \to \mathsf{K}_i\mathsf{K}_i\varphi$	4
$\langle \mathsf{K}_i \rangle \mathsf{K}_i\varphi \to \mathsf{K}_i\langle \mathsf{K}_i \rangle \varphi$	G

The axioms of the logic of belief KD45 include:

$\mathsf{B}_i(\varphi \to \psi) \to (\mathsf{B}_i\varphi \to \mathsf{B}_i\psi)$	K
$\mathsf{B}_i\varphi \to \langle \mathsf{B}_i \rangle \varphi$	D
$\mathsf{B}_i\varphi \to \mathsf{B}_i\mathsf{B}_i\varphi$	4
$\langle \mathsf{B}_i \rangle \varphi \to \mathsf{B}_i\langle \mathsf{B}_i \rangle \varphi$	5

(This is a technical companion to question 3 above.)

4. A pure modality is a finite sequence of zero or more of occurrences of $\Box$ and $\Diamond$, e.g., $-$, $\Box$, $\Diamond$, $\Box\Diamond$, $\Diamond\Diamond$, $\Diamond\Box$, $\Box\Diamond\Box$, etc. In *Modal Logic for Open Minds*, pp. 58-59, Johan van Benthem reports what H. B. Smith, a pioneer of modal logic, regarded as two plausible hypotheses in the 1930s:

   - *Distinction.* No two distinct pure modalities are provably equivalent.
   - *Comparison.* Given two pure modalities O_1 and O_2, one entails the other.

   Prove that no normal modal logic can accommodate both hypotheses. That is, if Σ is a normal modal logic, then *either* two distinct pure modalities O_1 and O_2 are equivalent, namely,

$$\Sigma \vdash \mathsf{O}_1\varphi \leftrightarrow \mathsf{O}_2\varphi,$$

   or there are incomparable pure modalities O_1 and O_2 such that

$$\Sigma \nvdash \mathsf{O}_1\varphi \to \mathsf{O}_2\varphi \quad \text{and} \quad \Sigma \nvdash \mathsf{O}_2\varphi \to \mathsf{O}_1\varphi.$$